It has an accuracy comparable to the existing dispersion models over a much wider range of parameters. The present model is open in nature, i.e., it can be adopted by any design engineer to achieve better accuracy in the model to match his measurement results for any specific substrate. The designer has to simply recalculate the A and B parameters against the experimental results and curve fit the data by linear or power regression, as presented in this paper. Moreover, the LDM is very simple and fast for computer-aided design (CAD) application and, with some modification, it could be adopted to model the dispersion in other planar transmission lines. The LDM is also suitable for effective presentation in classroom teaching.

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REFERENCES

[1] M. V. Schneider, "Microstrip dispersion," *Proc. IEEE*, vol. 60, pp. 144–146, Jan. 1972.
[2] W. J. Getsinger, "Microstrip dispersion model," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 34–39, Jan. 1973.
[3] M. Kobayashi, "Important role of inflection frequency in the dispersive properties of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 2057–2059, Nov. 1982.
[4] ——, "A dispersion formula satisfying recent requirement in microstrip CAD," vol. 36, pp. 1246–1250, Aug. 1988.
[5] P. Pramanick and P. Bhartia, "An accurate description of dispersion in microstrip," *Microwave, J.*, vol. 26, no. 12, pp. 89–92, Dec. 1983.
[6] E. Hammerstad and O. Jensen, "Accurate models for microstrip computer aided design," in *IEEE MTT-S Int. Microwave Symp. Dig.*, New York, NY, June 1980, pp. 407–409.
[7] E. Yamashita, K. Atsuki, and T. Veda, "An accurate dispersion formula of microstrip line for computer-aided design of microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 1036–1038, Dec. 1979.
[8] M. Kirschning and R. H. Jansen, "Accurate model for effective dielectric constant with validity up to millimeter-wave frequency," *Electron. Lett.*, vol. 18, pp. 272–273, Jan. 1982.
[9] H. A. Atwater, "Test for microstrip dispersion formula," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 619–621, Mar. 1988.
[10] E. F. Kuester and D. C. Chang, "An appraisal of methods for computation of the dispersion characteristics of open microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 691–694, July 1979.
[11] G. Kowalski and R. Pregla, "Dispersion characteristics of single and coupled microstrip," *AEU*, vol. 26, pp. 276–280, 1972.
[12] A. R. Van de Capelle, and P. J. Luypaert, "Fundamental and higher order modes in open microstrip lines," *Electron. Lett.*, vol. 9, pp. 345–346, 1973.
[13] E. Yamashita and K. Atsuki, "Analysis of microstrip like transmission lines by nonuniform discretization of integral equations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24. pp. 195–200, Apr. 1976.
[14] R. H. Jansen, "Unified user oriented computation of shielded, cover and open planar microwave and millimeter-wave transmission line characteristics," *Proc. Inst. Elect. Eng.*, vol. 3, no. 1, pp. 14–22, Jan. 1979.
[15] M. Kobayashi and F. Ando, "Dispersion characteristics of open microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 101–105, Feb. 1987.
[16] D. Mirshekar-Syahkal, *Spectral Domain Method for Microwave Integrated Circuits.* New York: Wiley, 1990, ch. 3.
[17] R. K. Hoffman, *Handbook of Microwave Integrated Circuits.* Norwood, MA: Artech House, 1987, ch. 3, p. 165.
[18] T. C. Edwards and R. P. Owens, "2-18 GHz dispersion measurement on 10–100-Ω microstrip lines on sapphire," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 506–513, 1976.
[19] Y. S. Lee, W. J. Getsinger, and L. R. Sparrow, "Barium tetratitanate MIC technology," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 655–660, July 1979.
[20] A. K. Verma and G. H. Sadr, "Unified dispersion model for multilayer microstrip line," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1587–1591, July 1992.
[21] E. Yamashita, "Variational method for the analysis of microstrip line transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 329–535, Aug. 1968.
[22] I. J. Bahl and R. Garg, 'Simple and accurate formulas for microstrip with finite strip thickness," *Proc. IEEE*, vol. 65, pp. 1611–1612, Nov. 1977.
[23] M. Kirschning, Ph.D. dissertation, Dept. Elect. Eng., Duisberg University, Duisberg, Germany, 1984.
[24] R. P. Owens, 'Predicted frequency dependence of microstrip characteristics impedance using the planar waveguide model," *Electron. Lett.*, vol. 12, no. 11, pp. 269–270, 1976.
[25] C. Shih, R. B. Wu, S. K. Jeng, and C. H. Chen, "Frequency-dependent characteristics of open microstrip lines with finite strip thickness," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 793–795, Apr. 1989.

New Empirical Unified Dispersion Model for Shielded-, Suspended-, and Composite-Substrate Microstrip Line for Microwave and mm-Wave Applications

A. K. Verma and Raj Kumar

***Abstract*—By introducing the concept of "virtual relative permittivity," this paper reports several closed-form dispersion models for a multilayered shielded/unshielded microstrip line over $1 < \epsilon_r \leq 20$, $0.1 \leq (w/h) \leq 10$, $(h_3/h) \geq 2$ in the frequency range up to 4 GHz · cm. The maximum deviation of the one model against the results of the spectral-domain analysis (SDA) is limited to 3%, while for the other three models, the maximum deviation is <2% and the root-mean-square (rms) deviation is <0.8%. This paper also reports improvement in the closed-form model of March for the determination of $\epsilon_{\text{eff}}(0)$ of the shielded microstrip line.**

***Index Terms*—Dispersion, multilayer microstrip.**

I. INTRODUCTION

The closed-form models are normally preferred by the designers due to their simplicity and ease in use. However, closed-form expressions for dispersion in the microstrip line on a composite/suspended substrate with and without a top shield are not available in the open literature. Jansen [1] has concluded that the dispersion modeling becomes extremely involved if the physical parameters of microstrip-like lines exceed four. Using the concept of the single-layer reduction (SLR) formulation, Verma and Hassani developed a unified dispersion model [2] for the shielded/unshielded multilayer microstrip line. However, this model degrades over the wider range of parameters. Replacement of the composite-substrate microstrip line by an equivalent permittivity of a single substrate has also been suggested by Finlay *et al.* [3]. However, no analytical method has been suggested by them for its determination.

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The authors are with the Department of Electronics Science, University of Delhi, New Delhi 110021, India.
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TABLE I
DEVIATION IN $\epsilon_{eff}(0)$ OF SHIELDED MICROSTRIP LINE AGAINST THE STATIC RESULTS OF SDA

ϵ_r	$\frac{W}{h}$	$\frac{h_2}{h_1}=2.0$				$\frac{h_2}{h_1}=6$			
		Var.	March	Mod.Mar.	Behl	Var.	March	Mod.Mar.	Behl
2.2	0.1	0.20	-0.32	0.35	0.45	-0.70	-0.18	-0.31	-0.34
	0.6	0.00	-0.42	0.20	0.54	-0.64	-0.47	-0.52	-0.52
	1.0	-0.24	-0.89	-0.29	0.41	-0.06	0.00	0.10	0.28
	5.0	-0.06	-2.56	-0.19	-0.23	-0.07	-0.53	-0.09	-0.11
	10.0	0.39	-4.16	-0.09	-1.41	1.07	-1.43	-0.65	-0.92
9.8	0.1	0.10	0.19	0.50	0.70	012	0.20	0.13	-0.18
	0.6	0.19	0.17	0.01	0.82	0.22	0.25	0.35	-0.45
	1.0	0.07	0.94	0.14	0.78	0.17	-0.03	0.18	-0.66
	5.0	-0.05	4.40	-0.72	-0.50	-0.08	-0.69	0.13	-0.68
	10.0	1.04	-7.47	0.29	-2.59	-1.09	-0.35	-0.83	-2.0
20.0	0.1	0.10	-0.30	0.06	0.51	0.12	0.21	0.19	-0.36
	0.6	0.08	-0.47	0.03	0.79	0.06	0.49	0.10	-0.26
	1.0	0.00	-1.04	0.13	0.68	0.15	0.09	0.32	-0.82
	5.0	-0.69	-4.74	0.18	-0.61	-0.11	-0.74	0.15	-0.89
	10.0	1.01	-8.18	0.23	-2.99	1.07	-2.19	-0.74	-2.16
40.0	0.1	0.10	-0.45	-0.23	0.61	0.19	-0.25	0.12	0.20
	0.6	0.20	-0.65	-0.13	0.58	0.28	-0.14	0.09	-1.18
	1.0	0.15	-0.99	0.22	0.72	0.27	-0.09	0.14	-1.17
	5.0	0.02	-4.88	0.23	-0.65	0.06	-0.72	0.19	-0.98
	10.0	0.49	-8.46	-0.22	-3.11	0.81	-2.39	-0.88	-2.41

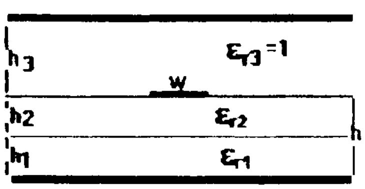

Fig. 1. Shielded composite substrate microstrip line.

II. STATIC EFFECTIVE RELATIVE PERMITTIVITY

The three-layer shielded microstrip line, shown in Fig. 1, can be reduced to the shielded ($h_2 \longrightarrow 0, \epsilon_{r2} = 1$), suspended ($h_3 \longrightarrow \infty, \epsilon_{r3} = \epsilon_{r1} = 1$) and composite ($h_3 \longrightarrow \infty, \epsilon_{r3} = 1$) substrate microstrip lines. Table I compares the percent deviation in $\epsilon_{\text{eff}}(0)$ for the shielded microstrip line calculated by the variational method (Var.) [4], expressions of March [5], and expressions of Behl [6] against the spectral-domain analysis (SDA) [7] at $f = 0.1$ GHz on $\epsilon_{r1} = 2.2, 9.8, 20$, and 40, for $(h_3/h_1) = 2$ and 6 at $(w/h_1) = 0.1$, 1, 5, and 10. Results have also been computed for $(h_3/h_1) = 3$, 4, 8, 10, and other ratios of (w/h_1) for comparison. The variational method has deviation $\leq$1%. The deviation in the models of March and Behl increases with nearness of the top shield and with increase in the relative permittivity ϵ_{r1} and (w/h_1) ratio. The model of March has deviation as high as 8.46% and the model of Behl has deviation $\leq$3.1%.

To improve the accuracy of the model of March, we have modified the expression for filling factor q (shielded) by introducing a correction factor K, which is a function of w/h_1 and h_3/h_1. A large number of cases were used to obtained numerical value of K and then a curve-fit expression was obtained. The expression for q' along with correction factor K is given by

$$q' = \tanh\left(0.922 + 0.121\left(1 + \frac{h_3}{h_1}\right) - 1.164\frac{h_1}{h_3}\right)K \tag{1}$$

where $K = 1 - \alpha((h_1/w)/(h_3/h_1)^{1.8})$

$$\begin{aligned} \alpha &= 0.0663 + 0.0576\frac{w}{h_1}, \qquad 0.1 \leq \frac{w}{h_1} \leq 0.6 \\ &= -0.2504 + 0.5684\frac{w}{h_1}, \qquad 0.6 \leq \frac{w}{h_1} \leq 3 \\ &= 10^y, \qquad 3 < \frac{w}{h_1} \leq 10 \end{aligned} \tag{2}$$

$$y = -0.49 + 1.485\log_{10}\left(\frac{w}{h_1}\right) \tag{3}$$

$$q(\text{shielded}) = 0.5 + q'(q(\text{unshielded}) - 0.5). \tag{4}$$

The q(unshielded) could be determined from the expressions of Hammerstad–Jansen [8]. Finally, $\epsilon_{\text{eff}}(0)$ of the shielded microstrip can be determined from

$$\epsilon_{\text{eff}}(0) = 1 + q(\text{shielded})(\epsilon_{r1} - 1). \tag{5}$$

Table I clearly shows that the modified model of March has deviation $\leq$0.9%. In most of the cases, error is $\leq$0.2%. As a matter of fact, the modified model of March has accuracy $\leq$0.9% even for $\epsilon_{r1} \leq 140$.

In the range $0.1 \leq (w/h) \leq 10, 0.2 \leq (h_1/h) < 0.8, 2.0 \leq \epsilon_{r2} \leq 20, h = h_1 + h_2$ for the suspended microstrip line, we have compared the results for $\epsilon_{\text{eff}}(0)$ determined by the variational-method and models of Tomar and Bhartia [9], Svacina [10], and Schellenberg [11] against the results of SDA Mirshekhar–Davies (SDA MD) [7] at frequency $f = 0.1$ GHz. The results of Tomar–Bhartia and Svacina give deviation from 4% to 36%. The model of Schellenberg has a deviation $\leq$2.12% for $\epsilon_{r2} \leq 13$, increasing up to 18% for $\epsilon_{r2} = 20$ at $(w/h) = 5, (h_1/h) = 0.2$. However, this is the best closed-form model which could be acceptable for computer-aided design (CAD) purposes. The $\epsilon_{\text{eff}}(0)$ of the composite substrate microstrip line could be determined by either the variational method or by the closed-form model of Svacina [10]. However, the deviation in the model of Svacina is 7.39%–18.7%, whereas the variational method has a deviation <1% for the suspended/composite substrate.

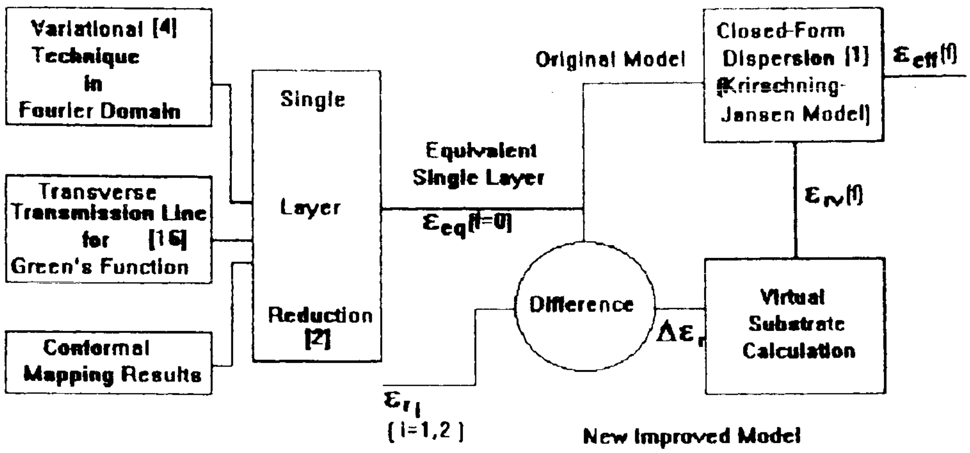

Fig. 2. Schematic diagram of unified dispersion model.

III. New Unified Dispersion Model

The modeling scheme of the previous unified dispersion model [2] and the present model is shown in Fig. 2. In the previous model, the multilayer microstrip line has been replaced by an equivalent single-layer substrate with a static equivalent relative permittivity $\epsilon_{\rm eq}$. This process is the SLR formulation. On the equivalent substrate with $\epsilon_{\rm eq}$, the Kirsching–Jansen (KJ) [12] dispersion model is used. Through extensive study of shielded-, suspended-, and composite-substrate microstrip lines, we have noticed that the effective relative permittivities of these structures do not move toward either the real permittivity or $\epsilon_{\rm eq}$ with increase in frequency. Instead, $\epsilon_{\rm eff}(f)$ appears to move toward a "virtual relative permittivity," which depends both upon the structure and operating frequency. The phrase "move toward $\epsilon_{rv}(f)$" indicates as if the virtual relative permittivity is a frequency-independent relative permittivity of the substrate having a fixed value. However, this is not true. The virtual relative permittivity could be viewed like the material dispersion in the substrate. For the shielded microstrip line at the lower end of the frequency, this virtual relative permittivity $\epsilon_{rv}(f)$ is higher than the ϵ_{r1}. It is also higher than $\epsilon_{\rm eq}$ of the composite/suspended microstrip line. The key issue in the new unified dispersion model is the empirical determination of $\epsilon_{rv}(f)$.

Fig. 3 shows the frequency-dependent nature of the virtual relative permittivity of the shielded-substrate ($\epsilon_{r1} = 9.8$), suspended-substrate ($\epsilon_{r1} = 1, \epsilon_{r2} = 9.8$), and composite substrate ($\epsilon_{r1} = 3.5, \epsilon_{r2} = 12.95, h = 0.2$ mm) microstrip lines. On investigating the behavior of $\epsilon_{rv}(f)$ and $d\epsilon_{\rm eff}(f)/df$ of various structures through use of the SDA, we have noticed that the virtual relative permittivity of the shielded microstrip line increases with the real relative permittivity of the substrate and the w/h_1 ratio, and decreases with an increase in the top-shield height. It also decreases with an increase in the frequency after a certain low frequency, which could be estimated from the frequency parameter f_p of Getsinger [13]. Likewise, the virtual relative permittivity of the suspended-/composite-substrate microstrip lines increases with increase in relative permittivity of the substrate, w/h ratio and operating frequency. The increase in $\epsilon_{rv}(f)$ is significant for the suspended microstrip line of $h_1/h_2 = 1$. The $\epsilon_{rv}(f)$ decreases with an increase in the air–gap/thickness of the low-permittivity substrate. Fig. 2 shows that the new unified dispersion model could be written as

$$\epsilon_{\rm eff}(f) = \epsilon_{rv}(f) - \frac{(\epsilon_{rv}(f) - \epsilon_{\rm eff}(0))}{1 + P(f)} \tag{6}$$

where $P(f)$ is a parameter obtained from the expressions of Krischning and Jansen [12]. In the present case, $P(f)$ is dependent upon $w/h, f \cdot h$ and $\epsilon_{rv}(f)$. For the shielded microstrip line $h = h_1$

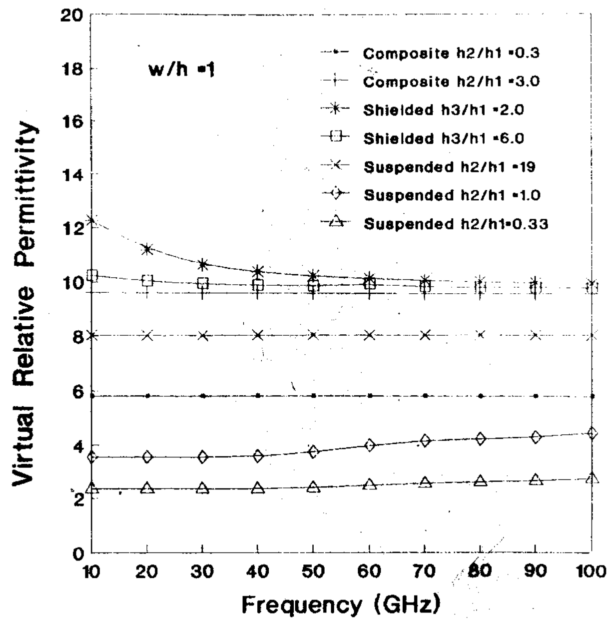

Fig. 3. Virtual relative permittivity.

and the composite/suspended substrate $h = h_1 + h_2$. $f \cdot h$ is in GHz · cm. In view of the above discussion, $\epsilon_{rv}(f)$ could be written as a combination of static and frequency-dependent parts of the relative permittivities.

$$\epsilon_{rv}(f) = \epsilon_{\rm eq} + \epsilon_{\rm radd}(f) \tag{7}$$

where, for the shielded microstrip line, $\epsilon_{\rm eq} = \epsilon_{r1}$ and for the suspended/composite-substrate microstrip line $\epsilon_{\rm eq}$ could be obtained from the SLR formulation [2]. The presence of top-shield or additional dielectric layer always alters the static equivalent relative permittivity $\epsilon_{\rm eq}$. However, with an increase in frequency, the field lines move toward the substrate of higher permittivities, bridging the gap between equivalent relative permittivity and real relative permittivity of higher value. Therefore, the $\epsilon_{\rm radd}(f)$ could be modeled around the difference relative permittivity, $\Delta\epsilon_r$. Thus, for the shielded microstrip line $\Delta\epsilon_r = \epsilon_{r1} - \epsilon_{\rm eq}$, for the composite ($\epsilon_{r2} > \epsilon_{r1}$)/suspended microstrip line $\Delta\epsilon_r = \epsilon_{r2} - \epsilon_{\rm eq}$, for the composite ($\epsilon_{r1} > \epsilon_{r2}$) microstrip line $\Delta\epsilon_r = \epsilon_{r1} - \epsilon_{\rm eq}$.

Satisfying the functional requirements discussed above, an expression for the additional relative permittivity could be written as follows:

$$\epsilon_{\rm radd}(f) = \sqrt{\gamma \Delta \epsilon_r}\,\frac{\left(1+\alpha \dfrac{w}{h}\right)}{H}(f_n)^{\beta}. \tag{8}$$

The parameters α, β, and γ control the influence of w/h ratio, operating frequency, and relative permittivity, respectively, on the $\epsilon_{\rm radd}(f) \cdot H$ is a parameter-controlling thickness of dielectric layers. For each specific structure, these parameters have been obtained by trial and error after comparing dispersion results of the present model against a large number of results obtained by the SDA MD over a wide range of physical parameters of the structures. The empirical relations for these parameters have been obtained by the curve fitting with the help of linear, power, exponential, and logarithmic regressions. f_n is the normalized frequency, which is $f_n = (f_p/\tau f)$ for the shielded microstrip and $f_n = (f/\tau f_p)$ for the suspended-/composite-substrate microstrip. The frequency parameter f_p for the individual structure is given by [13]

$$f_p = \frac{Z_o}{2\mu_o h\sqrt{(\epsilon_{\rm eff}(0))}} \tag{9}$$

where Z_o is the characteristic impedance of the microstrip line on the air–substrate, i.e., $\epsilon_{r1} = \epsilon_{r2} = \epsilon_{r3} = 1$. For the shielded microstrip line, Z_o could be obtained from the closed-form expressions of March [5], and for the open suspended/composite substrate, Z_o could be obtained from the closed-form expressions of Hammerstad–Jansen [8].

IV. Application of The New Unified Dispersion Model to Individual Structures

A. Shielded Microstrip Line

The virtual relative permittivity $\epsilon_{rv}(f)$ for this case is obtained by taking $H = (h_3/h_1)$, $\tau = 3$, $f_n = (f_p/3f)$, and $f_n = 1(f \le (f_p/3))$ in (8). The additional permittivity $\epsilon_{\rm radd}(f)$ for the shielded microstrip line meets the asymptotic requirements : $\epsilon_{\rm radd}(f) \longrightarrow 0, \epsilon_{rv}(f) \longrightarrow \epsilon_{r1}$ as $(h_3/h_1) \longrightarrow \infty$ and $\epsilon_{\rm radd}(f) \longrightarrow 0$, $\epsilon_{rv}(f) \longrightarrow \epsilon_{r1}$ as $f \longrightarrow \infty$. The expressions for α, β, and γ for the shielded microstrip line are summarized as follows:

$$\alpha = 1.4738\left(\frac{w}{h_1}\right)^{0.7758}, \qquad 0.1 \le \frac{w}{h_1} < 10 \tag{10}$$

$$\begin{aligned}\beta &= 1.5731 - 0.2308\left(\frac{w}{h_1}\right), && 0.1 \le \frac{w}{h_1} \le 0.6\\ &= 1.2604\left(\frac{w}{h_1}\right)^{-0.2535}, && 0.6 \le \frac{w}{h_1} < 5\\ &= 0.5, && 5 \le \frac{w}{h_1} \le 10\end{aligned} \tag{11}$$

$$\begin{aligned}\gamma &= -0.9869 + 1.1304\epsilon_{r1}, && 1.05 \le \epsilon_{r1} \le 2.2\\ &= 0.8228 + 0.3286\epsilon_{r1}, && 2.2 < \epsilon_{r1} < 12.95\\ &= -13.5 + 1.4285\epsilon_{r1}, && 12.95 \le \epsilon_{r1} \le 14\\ &= 0.3342 + 0.3542\epsilon_{r1}, && 14 < \epsilon_{r1} \le 20.\end{aligned} \tag{12}$$

The model has been tested in the range $(h_3/h_1) \ge 2$, $0.1 \le (w/h_1) \le 10$, $1.05 \le \epsilon_{r1} \le 20$. This model has a maximum deviation $\le$1.6% against the SDA MD [7].

B. Suspended Microstrip Line

The $\epsilon_{rv}(f)$ for this case is obtained by taking $\tau = 0.25$ and $f_n = (4f/f_p)$, $f_n = 1, (f \le f_p/4)$ in (8). The empirically determined parameters H, α, β, and γ are summarized as follows:

$$H = \exp\left(-5.1128\left(\frac{h_2}{h_1}\right)^{0.0544}\right) \tag{13}$$

$$\alpha = 1.5\exp-\left(\frac{w}{0.7213h}\right), \qquad h = h_1 + h_2 \tag{14}$$

$$\beta = 3.928(0.8118)^{A_1}[0.848(1.0248)^{A_1}]^{w/h}, \qquad A_1 = \frac{h_2}{h_1} \tag{15}$$

$$\gamma = 0.345A_2A_3(A_4)^{A_5}\left(\frac{\epsilon_{r2}}{6.0}\right)^{1.35}, \qquad 0.1 \le \frac{W}{h} \le 10. \tag{16}$$

For $0.6 \le (w/h) \le 4$, $A_2 = \exp(1.3911(0.4236)^{w/h} + 0.6861(w/h)^{-1.888} - 1)$.

For $4 < (w/h) \le 10, A_2 = 0.116(1.3474)^{w/h}$

$$A_3 = \frac{h_2}{2h_1}\left(\frac{h_2}{h_1} > 1\right), \qquad A_3 = 1 \text{ otherwise} \tag{17}$$

$$\begin{aligned}A_4 &= \frac{w}{h} && 0.6 \le \frac{w}{h} \le 5\\ A_4 &= \frac{h}{w}, && 5 < \frac{w}{h} \le 10\end{aligned} \tag{18}$$

$$\begin{aligned}A_5 &= 3.4266 - 4.2256\frac{w}{h}, && 0.1 \le \frac{w}{h} \le 0.6\\ &= 1.0153 - 0.2123\frac{w}{h}, && 0.1 \le \frac{w}{h} \le 5\\ &= -0.2339 + 0.0555\frac{w}{h}, && 5 < \frac{w}{h} \le 10.\end{aligned} \tag{19}$$

To improve the accuracy of the model for the suspended microstrip line, (8) for $\epsilon_{\rm radd}(f)$ is multiplied by a correction factor K' as follows:

$$K' = 1 - A_6\exp\left(-\left(\frac{f_1 - 60}{20}\right)^2\right) \tag{20}$$

where f_1 is in gigahertz. For $0.1 \le (w/h) \le 5$, $A_6 = A_7; ((h_2/h_1) \le 1)$ and $A_6 = (A_7/A_1); (1 < (h_2/h_1) \le 3)$. For $5 < (w/h) \le 10$, $A_6 = A_7; (0.05 \le (h_1/h_2) \le 3)$

$$\begin{aligned}A_7 &= -1.6938 + 0.3508\frac{w}{h}, && 0.1 \le \frac{w}{h} < 5\\ &= -0.25 + 0.05\frac{w}{h}, && 5 \le \frac{w}{h} \le 8\\ &= -0.825 + 0.125\frac{w}{h}, && 8 < \frac{w}{h} \le 10.\end{aligned} \tag{21}$$

The model has been tested for $1 < \epsilon_{r2} < 13, 0 \le (h_1/h_2) \le 3$, and $0.1 \le (w/h) \le 10$, $f \cdot h \le 2$ GHz $\cdot$ cm. It has a maximum deviation of 2% against the SDA MD at $(w/h) = 10, (h_2/h_1) = 1$. It has a root-mean-square (rms) deviation $\le$0.8%. The model meets the asymptotic requirements $\epsilon_{\rm radd}(f) \longrightarrow 0$, $\epsilon_{rv}(f) \longrightarrow 1$ as $h_2 \longrightarrow 0$ and $\epsilon_{\rm radd}(f) \longrightarrow 0$, $\epsilon_{rv}(f) \longrightarrow \epsilon_{r2}$ as $h_1 \longrightarrow 0$.

C. Composite Substrate

It has been modeled for two different cases, namely, $\epsilon_{r2} > \epsilon_{r1}$ and $\epsilon_{r1} > \epsilon_{r2}$. The case $\epsilon_{r2} > \epsilon_{r1}$ could also be used for the suspended structure with limited range of parameters, i.e., for $h_1 \le (h_2/19)$ and $h_1 \ge 3h_2$ and $2 \le \epsilon_{r2} \le 6$. For $\epsilon_{r2} > \epsilon_{r1}$, the $\epsilon_{\rm eff}(f)$ is obtained by taking $\tau = 1$, $f_n = (f/f_p)$ $(f > f_p)$, $f_n = 1$ $(f \le f_p)$ and $\alpha = 0.6$ in (8). The curve-fitted expression for H, β, and γ are as follows:

$$H = \exp\left(-4.4292\left(\frac{h_2}{h_1}\right)^{0.1115}\right) \tag{22}$$

$$\beta = C_1\left(\frac{w}{h}\right)^{-C_2}, \qquad h = h_1 + h_2 \tag{23}$$

$$\begin{aligned}C_1 &= 2.6888 + 0.3566\left(\frac{h_2}{h_1}\right), && \frac{1}{3} \le \frac{h_2}{h_1} \le 1\\ &= 3.4093 - 0.3639\left(\frac{h_2}{h_1}\right), && 1 < \frac{h_2}{h_1} < 3\\ &= 2.8959 - 0.1261\left(\frac{h_2}{h_1}\right), && 3 \le \frac{h_2}{h_1} \le 19\end{aligned} \tag{24}$$

$$C_2 = 0.3573 - 0.05745\left(\frac{h_2}{h_1}\right), \qquad \frac{1}{3} \le \frac{h_2}{h_1} \le 1$$
$$= 0.3375 - 0.0377\left(\frac{h_2}{h_1}\right), \qquad 1 < \frac{h_2}{h_1} \le 3$$
$$= 0.266 - 0.014\left(\frac{h_2}{h_1}\right), \qquad 3 < \frac{h_2}{1} \le 19 \tag{25}$$
$$\gamma = 0.3285(1.2012)^{\epsilon_{r2}}, \qquad 2 < \epsilon_{r2} < 9.8$$
$$= 0.0655(1.4173)^{\epsilon_{r2}}, \qquad 9.8 \le \epsilon_{r2} < 13. \tag{26}$$

This model has been tested against the SDA MD for $2 \le \epsilon_{r2} \le 13$, $(\epsilon_{r2}/\epsilon_{r1}) \le 5$, $0.1 \le (w/h) \le 10$, $0 \le (h_1/h_2) \le 3$, and $f \cdot h \le 4$ GHz $\cdot$ cm. This model has maximum deviation of 2% at $(w/h) = 10$, ($h_2/h_1 = 1$, $\epsilon_{r1} = 3.5$, and $\epsilon_{r2} = 12.95$). The rms deviation in the model is $\le$1.8%.

For $\epsilon_{r1} > \epsilon_{r2}$, the $\epsilon_{\text{eff}}(f)$ is obtained by taking $H = 2.0(h_2/h_1)$, $\tau = (1/2)$, $f_n = (2f/f_p)$, $f_n = 1$ $(f \le (f_p/2))$, and $\alpha = 1$ for $0.1 \le (w/h) \le 10$, $0.01 \le (h_2/h_1) \le 0.05$ in (8). This case is applicable for a thin passivation layer. The expression for γ is the same as (26). The curve-fitted expression for β is as follows:

$$\beta = 0.8028\left(\frac{w}{h}\right)^{-0.4288}, \qquad h = h_1 + h_2. \tag{27}$$

In this case, the model has a maximum deviation of 2% against the SDA MD for $f \cdot h \le 4$ GHz $\cdot$ cm.

D. Shielded Composite-Substrate Microstrip Line

For $\epsilon_{r1} > \epsilon_{r2}$, the $\epsilon_{\text{eff}}(f)$ of this is obtained by taking the following parameters in (8):

$$H = \sqrt{\frac{h_3}{h}}, \qquad h = h_1 + h_2. \tag{28}$$

For $f \cdot h \le 0.8$ GHz $\cdot$ cm, $f_n = f_p/(2.5(f - \exp(0.035f)))$.

For $0.8 \le f \cdot h \le 2$ GHz $\cdot$ cm, $f_n = f_p/(2.0(f - \exp(0.035f)))$.

For $2 \le f \cdot h \le 6$ GHz $\cdot$ cm, $f_n = (f_p/133.78)$, where f and f_p are in gigahertz, $\beta = \gamma = 0.5$, and the empirical expression for α is as follows:

$$\alpha = C_3\left(\frac{w}{h}\right)^{-C_4}, \qquad 0.6 \le \frac{w}{h} \le 10 \tag{29}$$
$$C_3 = 2.7392 - 14.587\left(\frac{h_2}{h_1}\right), \qquad \frac{h_2}{h_1} \le 0.05$$
$$= 1.9715, \qquad 0.05 \le \frac{h_2}{h_1} \le 0.33$$
$$= 2.1986 - 0.6821\left(\frac{h_2}{h_1}\right), \qquad 0.33 \le \frac{h_2}{h_1} \le 1 \tag{30}$$
$$C_4 = 0.4758 - 0.9732\left(\frac{h_2}{h_1}\right), \qquad \frac{h_2}{h_1} \le 0.05$$
$$= 0.4246, \qquad 0.05 < \frac{h_2}{h_1} \le 0.33$$
$$= 0.4378 - 0.03959\left(\frac{h_2}{h_1}\right), \qquad 0.33 < \frac{h_2}{h_1} \le 1. \tag{31}$$

The $\epsilon_{rv}(f)$ meets the asymptotic requirement with respect to the top shield height, i.e., $\epsilon_{rv}(f) \longrightarrow \epsilon_{\text{eq}}$ for $(h_3/h) \longrightarrow \infty$. For $(h_3/h) > 8$, the model could also be used for the determination of the dispersion behavior of the open composite-substrate microstrip line. For $\epsilon_{r2} > \epsilon_{r1}$, the $\epsilon_{\text{eff}}(f)$ is obtained by taking the following parameters in (8):

$$H = \sqrt{\frac{h_3}{h}}, \qquad h = h_1 + h_2 \tag{32}$$

for $f \cdot h \le 0.8$ GHz $\cdot$ cm, $f_n = (f_p/2.2(f - \exp(0.035f)))$;
for $0.8 \le f \cdot h \le 1.2$ GHz $\cdot$ cm, $f_n = (f_p/2(f - \exp(0.035f)))$;
for $1.2 \le f \cdot h \le 2.8$ GHz $\cdot$ cm, $f_n = (f_p/125)$.

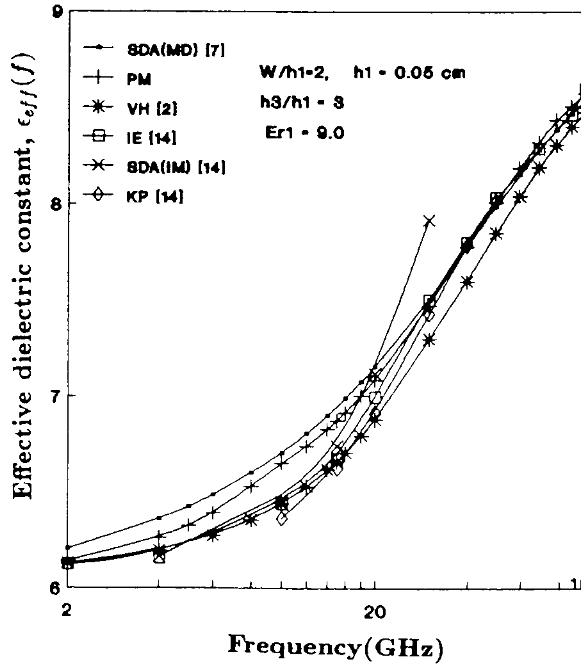

Fig. 4. Comparison of present dispersion model (PM) for the shielded microstrip line against various methods.

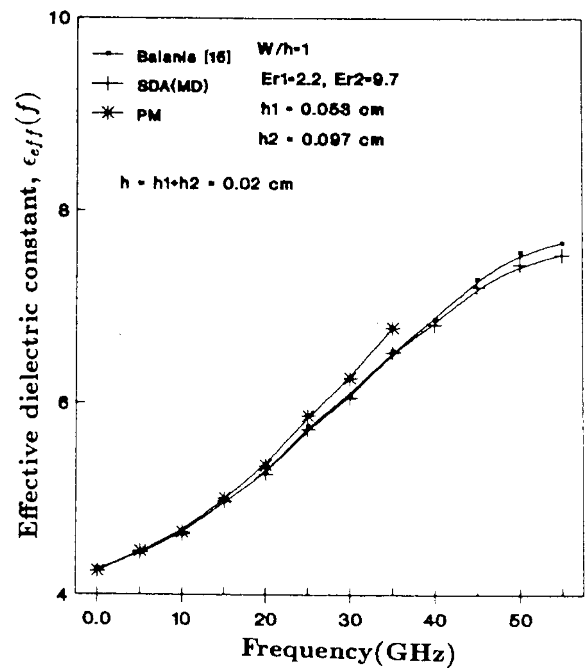

Fig. 5. Comparison of dispersion results of present model (PM) against the SDA from two sources.

Again, f and f_p are in gigahertz. The empirical expressions for parameters α, β, and γ for this case are as follows:

$$\alpha = C_5\left(\frac{w}{h}\right)^{-C_6}, \qquad 0.1 \le \frac{w}{h} \le 10 \quad 1 \le \frac{h_2}{h_1} \le 19 \tag{33}$$
$$C_5 = 0.6431\left(\frac{h_2}{h_1}\right)^{-0.2506} \qquad C_6 = 0.2977\left(\frac{h_2}{h_1}\right)^{-0.02651} \tag{34}$$
$$\beta = 0.5, \gamma = \frac{1}{2\left(1 + C_7\frac{h_2}{h_1}\right)}, \qquad C_7 = 1.4134\left(\frac{h_2}{h_1}\right)^{-1.6936}. \tag{35}$$

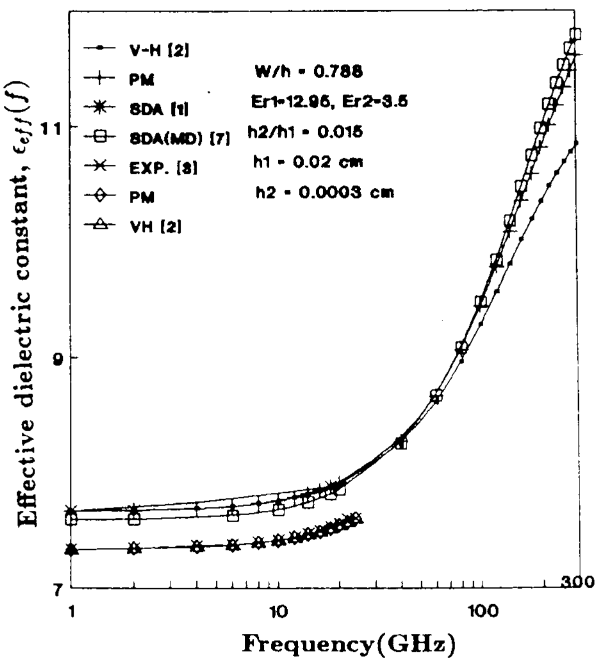

Fig. 6. Dispersion of composite microstrip line.

For the case $\epsilon_{r1} > \epsilon_{r2}$, the model has a maximum deviation of 2% for $\epsilon_{r1} = 12.95$, $\epsilon_{r2} = 3.5 f \cdot h \leq 4$ GHz · cm, $0.1 \leq (w/h) \leq 10$, $(h_3/h) \geq 2$. For $\epsilon_{r2} > \epsilon_{r1}$, $0 \leq (h_2/h_1) \leq 3$, $(h_3/h_1) \geq 2$, the model has a maximum deviation of 3% for $f \cdot h \leq 2.6$ GHz · cm.

V. Comparison to Published Results

The results of the present model have been thoroughly compared against the SDA MD [7]. The seventh-order Legendre polynomial showing good convergency has been used as the basis functions in the SDA MD formulation. However, the detailed comparison of results is not presented due to a lack of space. We have already clearly presented the percent deviation and range of parameters for each model. While discussing the accuracy of the models, we should keep in mind that the present model utilizes the Krischning–Jansen [5] dispersion model for the open microstrip line, which has deviation ≤2%, for $0.1 \leq (w/h) \leq 100$, $1 \leq \epsilon_r \leq 20$, $0 \leq f \cdot h \leq 3.9$ GHz · cm. Fig. 4 further compares the dispersion results of the present model for the shielded microstrip line against several full-wave methods. The full-wave results have been taken from [14, Fig. 5]. The full-wave results differ among themselves with a deviation as high as 6.7%. This high deviation is perhaps due to the selection of basis functions. The previous unified dispersion model of Verma–Hassani has a deviation ≤4%, whereas the present model has a deviation ≤1%. Fig. 5 further compares the composite substrate case $(\epsilon_{r2} > \epsilon_{r1})$ against the results of the SDA from Balanis [15] for substrate $\epsilon_{r1} = 2.2$, $\epsilon_{r2} = 9.7$, $(w/h) = 1$. For $f \cdot h = 3.9$ GHz · cm, the present model has a deviation of only 1.8% against the SDA of Balanis. For the case $\epsilon_{r1} > \epsilon_{r2}$ $(\epsilon_{r1} = 12.95$, $\epsilon_{r2} = 3.5)$, Fig. 6 further compares the dispersion models and the SDA MD up to 300 GHz and against the experimental results up to 24 GHz. The maximum deviation in the model is 0.36%. The results of the SDA adopted by Jansen [1] at 2 and 18 GHz are also included for comparison. While comparing the dispersion models against the experimental results, we should keep in view that the fabricational process and the substrate could have variation by ±3%, which results into an error in the computed $\epsilon_{\text{eff}}(0)$ above 3% for the multilayer structure.

VI. Conclusion

We have presented a unified dispersion model to achieve an accuracy better than ≤2% for the dispersion in various microstrip-like structures. Our modeling is satisfactory for the first three cases. However, an improvement is needed for the shielded composite-substrate microstrip line where the deviation is ≤3% for $f \cdot h \leq 2.6$ GHz · cm. The concept of virtual relative permittivity could be further extended for the dispersion modeling of the multilayer shielded/unshielded coupled microstrip line.

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References

[1] R. H. Jansen, "A novel CAD tool and concept compatible with the requirement of multilayer GaAs MMIC technology," in *IEEE MTT-S Symp. Dig.*, June 1985, pp. 711–714.
[2] A. K. Verma and G. H. Sadr, "Unified dispersion model for multilayer microstrip line," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1887–1891, July 1992.
[3] H. J. Finlay, R. H. Jansen, J. A. Jenkins, and I. G. Eddison, "Accurate characterization and modeling of transmission lines for GaAs MMIC," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 961–967, June 1988.
[4] E. Yamashita, "Variational method for the analysis of microstrip-like transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 529–535, Aug. 1968.
[5] S. Mar., "Microstrip packaging watch the last step," *Microwaves*, vol. 20, no. 12, pp. 83–94, 1981.
[6] J. Behl, "Use exact method for microstrip design," *Microwaves*, vol. 17, no. 12, pp. 61–62, Dec. 1978.
[7] D. Mirshekar-Syahkal and J. B. Davies, "Accurate solution of microstrip and coplanar structures for dispersion and dielectric and conductor losses," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 694–699, July 1979.
[8] E. Hammerstad and O. Jensen, "Accurate models for microstrip computer-aided design." in *IEEE MTT-S Symp. Dig.*, New York, NY, June 1980, pp. 190–192.
[9] R. S. Tomar and P. Bhartia, "New quasi-static models for the computer-aided design of suspended and inverted microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 453–457, Apr. 1987.
[10] J. Svacina, "Analysis of multilayer lines by a conformal mapping method," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 769–772, Apr. 1992.
[11] J. M. Schellenberg, "CAD models for suspended and inverted microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-43, pp. 1247–1252, June 1995.
[12] M. Krischning and R. H. Jansen, "Accurate model for effective dielectric constant with validity up to millimeter-wave frequency," *Electron. Lett.*, vol. 18, pp. 272–273, Jan. 1982.
[13] W. J. Getsinger, "Microstrip dispersion model," *IEEE Trans. Microwave Theory Tech.*, vol. 21, pp. 34–39, Jan. 1973.
[14] E. Yamashita and A. Atsuki, "Analysis of microstrip like transmission lines by nonuniforn discretization of integral equations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 195–200, Apr. 1976.
[15] J. P. Gilb and C. A. Balanis, "Pulse distortion on multilayer coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1620–1627, Oct. 1989.
[16] R. Cramapgane, M. Ahmadpanah, and J. L. Guiraud, "A simple method for determining the Green's function for a large class of MIC lines having multilayered dielectric structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 82–87, Feb. 1978.